Inconsistency and Over-Determination in Balance of Payments Constrained Growth Models: a note

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1 – Introduction

The objective of this short note is to revaluate the solution given by Palley (2002) for the inconsistency problem in BOP constrained growth models. As we shall argue in the next paragraphs, making income elasticity of imports a positive function of capacity utilization (or a negative function of excess capacity) is not enough condition to eliminate the over-determination problem from the structure of BOP models. The missing element in Palley´s solution is the failure in distinguishing warrant from natural rate of growth. Once we distinguish between these concepts, the problem of over-determination appears again: the rate of capacity utilization that is compatible with balanced growth will be, in general, different from the level of capacity utilization that is compatible with current account balance.

In order to solve this over-determination problem, it is necessary to introduce the level of real exchange rate in the formal structure of BOP constrained growth models. This can be done if one considers income elasticity of imports as a function of the level of productive specialization of home country which depends on the level of real exchange rate. Once we make this change in the formal structure of BOP constrained growth models, than over-determination problem vanishes and a consistent long-run equilibrium position can be formally calculated.

The solution proposed for the over-determination problem of BOP constrained growth models has the role of putting real exchange rate at the center of Post-Keynesian Growth Models, since now real exchange rate is the variable that is responsible for making compatible the requirement of current account balance with the requirement of equality between the growth rate of real output and the growth rate of potential output.

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2 – A Modified Version of BOP Constrained Growth Model.

Conventional versions of BOP growth model take for granted that capitalist economies are constrained by effective demand, not by supply factors, in the long run. This assumption is supposed to be a sufficient condition for ignoring any element of the supply side of the economy in the structure of these models. In particular, it is supposed that capital stock always adjust itself to the growth rate of real output that is given by BOP restrictions.

As it is shown by Palley (2002, p.120), the failure to incorporate the supply side of the economy in the structure of BOP models give rise to an internal inconsistency due to the fact that, in the long run, not only is growth constrained by the requirement of current account balance, but also the rate of growth of real output must be equal to the rate of growth of productive capacity in order to allow a constant rate of capacity utilization. According to Palley this inconsistency could be solved by making the income elasticity of demand for imports to be a negative function of excess capacity. By doing so, growth rate of real output will be determined by the requirement that demand growth has to be equal to growth rate of productive capacity. The BOP restraint will then determine the level of excess capacity, given the growth rate of real output.

The solution given by Palley, however, do not make a distinction between the growth rate of productive capacity – given by the growth rate of capital stock – and the growth rate of potential output – given by the sum of productivity growth and the growth rate of the labor force. In other words, Palley take for granted that natural and warrant rate of growth are necessarily equal. If this equality do not hold, then the BOP growth model is over-determined in the sense that one variable (capacity utilization) is determined by two different and independent equations, assuming values that are, in general, inconsistent one with the other.

Let us consider an open economy described by the following equations:

$$\hat{X} = a_0 g^* \quad (1)$$

$$\hat{M} = (b_0 + b_1 u)g \quad (2)$$

$$\hat{X} = \hat{M} \quad (3)$$

$$\hat{u} = g - g_k \quad (4)$$

$$g_k = g_0 + g_1 u \quad (5)$$

$$g_n = \tau + n \quad (6)$$

$$\tau = c_0 + c_1 g \quad (7)$$

Where: $\hat{X}$ is the growth rate of real exports, $\hat{M}$ is the growth rate of real imports, g^* is the growth rate of world´s real income, g is the growth rate of domestic real output, $\hat{u}$ is the growth rate of capacity utilization, g_k is the growth rate of capital stock, g_n is the natural growth rate, n is the growth rate of labor force, τ is the growth rate of labor productivity.

Equations (1) and (2) are, respectively, export and import growth equations. Equation (3) is the balance of payments constraint in a setting where there are no capital flows to finance any current account deficit. Equation (5) gives the growth rate of capital stock as a positive function of capacity utilization due to the well known accelerator effect. Equation (6) is the definition of natural growth rate and equation (7) express the so called Kaldor-Verdoorn´s Law, according to which productivity growth is a positive function of the growth rate of real output.

Getting (7) in to (6) we have:

$$g_n = c_0 + n + c_1 g \quad (8)$$

Balanced-growth requires the equality between warrant and natural growth rates, that is:

$$g_k = g_n \quad (9)$$

Besides that, it is also necessary that capacity utilization is constant trough time:

$$\hat{u} = 0 \leftrightarrow g_k = g \quad (10)$$

Substituting (5) and (10) in (4), we get:

$$g = g_0 + g_1 u \quad (11)$$

After substituting (9) and (10) in (8) we get:

$$g = \frac{c_0+n}{1-c_1} \quad (12)$$

Equation (12) gives the growth rate of real output that is compatible with the equality of the warrant and natural growth rate. The mechanism by which warrant and natural growth rates adjust one to the other is the level of capacity utilization. Indeed, substituting (12) in (11) we get:

$$u = \left(\frac{1}{g_1}\right)\left[\frac{c_0+n}{1-c_1} - g_0\right] \quad (13)$$

Equation (13) gives the value of capacity utilization that is compatible with the equality between warrant and natural growth rates, that is, the rate of capacity utilization that allows productive capacity to grow at the same rate of potential output.

This is not, however, the only value for capacity utilization that is determined by the model. Indeed, after substituting (1) and (2) in (3) we get:

$$a_0 g^* = (b_0 - b_1 u)g \quad (14)$$

Finally, after substituting (12) into (14), we arrive at:

$$u = \left(\frac{1}{b_1}\right)\left[a_0 g^* \left(\frac{1-c_1}{c_0+n}\right) - b_0\right] \quad (15)$$

Equation (15) gives the value of capacity utilization that is compatible with the BOP constraint. As we can easily see, the level of capacity utilization that is compatible with BOP constraint is, in general, different from the level of capacity utilization that is compatible with a balanced growth.

This means that there is a problem of over-determination in the system; i.e one variable is determined by two different and independent equations. So, we can conclude that making income elasticity of imports a positive function of capacity utilization will not solve the inconsistency of BOP model, if this model is correctly specified.

3 – Solving the Inconsistency: Real Exchange Rate and Income Elasticity of Imports.

Although Palley´s solution is not able to eliminate the inconsistency problem in BOP constrained growth models, it is on the right track. In order to eliminate the over-determination of a system of equations, it is necessary to increase the number of endogenous variables, which requires transform some parameters into unknowns of the system. The relevant question is which variable should be considered as an endogenous variable in order to eliminate the over-determination problem?

A remarkable omitted variable in BOP constrained growth models is the level of real exchange rate. This omission is justified by the assumption that in dynamic models what matters is not the level of real exchange rate, but the rate of change of this variable. Being so, the rate of change of real exchange rate could, in principle, influence the growth rates of imports and exports through price elasticities in import and export dynamic equations. However, a non-zero rate of change for real exchange rate is, by definition, incompatible with balanced growth. This means that for the calculation of the balanced growth rate of real output, rate of change of real exchange rate should be set in zero, eliminating all influence of real exchange rate from the system.

There is a channel by which the level of real exchange rate could influence the growth rate of exports and imports that is not considered in the traditional BOP constrained growth literature. This channel consists in making income elasticities of imports and/or exports a function of real exchange rate.

Income elasticities are dependent upon the productive structure of the economy, more specifically, on the *level of specialization* of its productive structure. A high level of specialization is associated with a high marginal propensity to import and, consequently, with a high income elasticity of imports. It is also clear that a high level of specialization will be associated with low elasticity of exports, since the economy will have few different types of goods to export in face of increasing world demand.

The level of specialization of an economy is affected by real exchange rate, since this variable is of fundamental importance for determining unitary labor costs through out the world and, consequently, the worldwide level of productive specialization. In this setting, a higher (more depreciated) real exchange rate will induce a decrease in productive specialization, since it will reduce unitary labor costs in domestic economy, making a large number of goods be profitably produced in home country. This change in the level of productive specialization will induce a decrease in the income elasticity of imports and, probably, also an increase in the income elasticity of exports[1].

In face of these considerations we will introduce the level of real exchange rate in the structure of a BOP growth model. Real exchange rate will affect (negatively) the value of income elasticity of imports, but also the growth rate of capital stock. This effect is due to the fact that a real exchange rate devaluation is associated with increases in profit margins and, consequently, in the share of profits in real income. Supposing, as Bhaduri and Marglin (1990), that investment is a separable function of capacity utilization and profit share, than real exchange rate can be introduced in investment function as a *proxy* for profit share.

The new version of BOP constrained growth model is given by:

$$\hat{X} = a_0 g^* \quad (1)$$

$$\hat{M} = (b_0 - b_1\theta)g \quad (2a)$$

$$\hat{X} = \hat{M} \quad (3)$$

$$\hat{u} = g - g_k \quad (4)$$

$$g_k = g_0 + g_1 u + g_2\theta \quad (5a)$$

$$g_n = \tau + n \quad (6)$$

$$\tau = c_0 + c_1 g \quad (7)$$

Where: θ is the level of real exchange rate.

In the system of equations shown above there are only two new equations. The first one is equation (2a) where income elasticity of imports is now supposed to be a negative function of real exchange rate. The second is equation (5a) where real exchange rate is added as a new determinant of the growth rate of capital stock as a *proxy* for profit share.

As in the model presented in section 2, the balanced growth rate of real output is given by:

$$g = \frac{c_0 + n}{1 - c_1} \quad (12)$$

[1] For empirical evidence regarding the relation between income elasticity of exports and real exchange rate see Oreiro et al (2012).

Putting (12) into (5a) we get:

$$u = \left(\frac{1}{g_1}\right)\left\{\left[\left(\frac{c_0+n}{1-c_1}\right) - g_0\right] - g_2\theta\right\} \quad (16)$$

Equation (16) gives the level of capacity utilization that is compatible with balanced growth as function of real exchange rate. It is easy to shown that:

$$\frac{\partial u}{\partial \theta} = -\left(\frac{g_2}{g_1}\right) < 0 \quad (16a)$$

From equations (1), (2a), (3) and (12) we have:

$$\theta = \left(\frac{1}{b_1}\right)\left\{b_0 - a_0 g^* \left(\frac{1-c_1}{c_0+n}\right)\right\} \quad (17)$$

Equation (17) gives the value of real exchange rate that adjusts the growth rate compatible with balance of payments constrained growth with the growth rate compatible with balanced growth. This means that the role of real exchange rate in BOP constrained growth models in to make compatible the requirement of current account balance with the requirement that the growth rate of real output to be equal to the growth rate of potential output. Real exchange rate is now at the center of Post Keynesian growth models.

Once real exchange rate is determined in equation (17), the level of capacity utilization could be determined by equation (16). Than we have:

$$u = \left(\frac{1}{g_1}\right)\left\{\left[\left(\frac{c_0+n}{1-c_1}\right) - g_0\right] - \left(\frac{g_2}{b_1}\right)\left[b_0 - a_0 g^* \left(\frac{1-c_1}{c_0+n}\right)\right]\right\} \quad (18)$$

The determination of the level of capacity utilization can be visualized by figure 1 below:

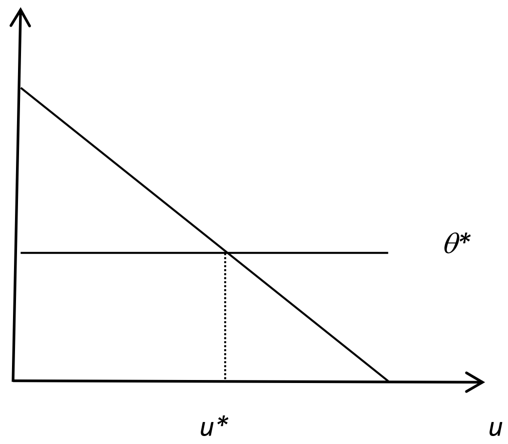

Figure 1

Equations (12), (17) and (18) determine the growth rate of real output, the level of real exchange rate and the level of capacity utilization that are simultaneously compatible with balanced growth and balance of payments constraint. There is no more inconsistency or over-determination in BOP constrained growth models.

4 – Final Remarks.

The solution proposed here for the over-determination problem of BOP constrained growth models has two important features. The first one is that it was able to integrate in a unified and common framework some elements of Kaldorian and Kaleckian growth models. From Kaldorian growth models we have the idea that natural rate of growth is endogenous and dependent on the parameters of the "Technical Progress Function". From Kaleckian growth models we have the idea that capacity utilization is also an endogenous variable, dependent on effective demand. Indeed, in the model develop here the growth rate of exports – determined by the product between income elasticity of exports and the growth rate of world income – is an important determinant of capacity utilization.

The second one is to show that in the long-run equilibrium, there is no such a thing as an external constraint for growth. In fact, if real exchange rate is at its proper level, income elasticity of imports will assume a value that allows imports to grow at a rate compatible with current account balance. Growth of domestic output will only be limited by the level of dynamic economies of scale, which determines the value of parameter c_1 in the Kaldor-Verdoorn´s equation. The level of dynamic economies of scale depends, among many other variables, on the share of manufacturing industry in GDP (See Botta, 2009). This means that balanced growth rate is a positive function of manufacturing share in GDP, indicating that manufacturing industry is the engine of economic growth in the long run.

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